

Week #11

Some problems and solutions selected or adapted from Stewart Calculus.

Triple Integrals

1. Evaluate the triple integral

(a) $\iiint_E 6xy \, dV$, where E lies under the plane $z = 1 + x + y$ and above the region in the xy -plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

(b) $\iiint_E x \, dV$, where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$.

2. Use a triple integral to find the volume of the solid enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$.

3. Express the integral $\iiint_E f(x, y, z) \, dV$ as an iterated integral in six different ways, where E is the solid bounded by $y = 4 - x^2 - 4z^2$, $y = 0$.

Triple Integrals in Cylindrical Coordinates

4. Evaluate $\iiint_E (x + y + z) \, dV$, where E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$.

5. Evaluate $\iiint_E x^2 \, dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$,

and below the cone $z^2 = 4x^2 + 4y^2$.

6. Find the volume of the solid that is enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$.

7. Find the mass and center of mass of the solid S bounded by the paraboloid $z = 4x^2 + 4y^2$ and the plane $z = a$ ($a > 0$) if S has constant density K .