

Week #10

Some problems and solutions selected or adapted from Stewart Calculus.

Double Integrals in Polar Coordinates

- Evaluate the given integral by changing to polar coordinates.
 - $\iint_D x^2 y \, dA$, where D is the top half of the disk with center the origin and radius 5
 - $\iint_R \sin(x^2 + y^2) \, dA$, where R is the region in the first quadrant between the circles with center the origin and radii 1 and 3
 - $\iint D e^{-x^2 - y^2} \, dA$, where D is the region bounded by the semicircle $x = \sqrt{4 - y^2}$ and the y -axis
 - Use a double integral to find the area of the region inside one loop of the rose $r = \cos 3\pi$.
 - Use polar coordinates to find the volume of the given solid.
 - Under the cone $z = \sqrt{x^2 + y^2}$ and above the disk $x^2 + y^2 \leq 4$
 - Inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$
 - Evaluate the iterated integral by converting to polar coordinates.
 - $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \sin(x^2 + y^2) \, dy \, dx$
 - $\int_0^1 \int_y^{\sqrt{2-y^2}} (x + y) \, dx \, dy$
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Applications of Double Integrals

- Find the mass and center of mass of the lamina that occupies the region D and has the given density function ρ .
 - $D = \{(x, y) \mid 1 \leq x \leq 3, 1 \leq y \leq 4\}$; $\rho(x, y) = ky^2$
 - D is bounded by $y = 1 - x^2$ and $y = 0$; $\rho(x, y) = ky$
- A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant. Find the center of mass if the density at any point is proportional to the distance from the x -axis.
- The boundary of a lamina consists of the semicircles $y = \sqrt{1 - x^2}$ and $y = \sqrt{4 - x^2}$ together with the portions of the x -axis that join them. Find the center of mass of the lamina if the density at any point is proportional to its distance from the origin.