Week #9

Some problems and solutions selected or adapted from Stewart Calculus.

Iterated Integrals

1. Calculate the iterated integral.

   (a) $\displaystyle\int_1^4 \int_0^2 (6x^2y - 2x)\, dy\, dx$

   (b) $\displaystyle\int_0^1 \int_0^1 \nu(u + \nu^2)^4\, du\, d\nu$

2. Calculate the double integral.

$$\iint_R \frac{xy^2}{x^2+1}\, dA$$

   where $R = \{(x, y) \mid 0 \le x \le 1, -3 \le y \le 3\}$.

3. Find the volume of the solid that lies under the plane $4x + 6y - 2z + 15 = 0$ and above the rectangle $R = \{(x, y) \mid -1 \le x \le 2, -1 \le y \le 1\}$.

Double Integrals over General Regions

4. Evaluate the double integral.

   (a) $\displaystyle\iint_D y^2\, dA$,
   $D = \{(x, y) \mid -1 \le y \le 1, -y - 2 \le x \le y\}$

   (b) $\displaystyle\iint_D x\, dA$, $D = \{(x, y) \mid 0 \le x \le \pi, 0 \le y \le \sin x\}$

5. Find the volume of the solid bounded by the coordinate planes and the plane $3x + 2y + z = 6$.

6. Sketch the region of integration and change the order of integration.

   (a) $\displaystyle\int_0^1 \int_0^y f(x, y)\, dx\, dy$

   (b) $\displaystyle\int_1^2 \int_0^{\ln x} f(x, y)\, dy\, dx$

7. Evaluate the integral by reversing the order of integration.

   (a) $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy\, dx$

   (b) $\displaystyle\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x}\, dx\, dy$