Week #8

Some problems and solutions selected or adapted from Stewart Calculus.

Taylor and Maclaurin Series

1. Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin series. [Assume that f has a power series expansion. Do not show that $R_n(x) \to 0$.]
   (a) $f(x) = (1-x)^{-2}$
   (b) $f(x) = 2^x$

2. Find the Taylor series for $f(x)$ centered at the given value of a. [Asume that f has a power series expansion. Do not show that $R_n(x) \to 0$.]
   (a) $f(x) = x^4 - 3x^2 + 1$, $a = 1$
   (b) $f(x) = \ln x$, $a = 2$

3. Use a Maclaurin series in Table 1 to obtain the Maclaurin series for the function.

$$f(x) = \frac{x}{\sqrt{4+x^2}}$$

4. Use the Maclaurin series for $\cos x$ to compute $\cos 5^\circ$ correct to five decimal places.

5. Use series to approximate the definite integral to within the indicated accuracy.
   (a) $\displaystyle\int_0^{1/2} x^3 \arctan x \, dx$ (four decimal places)
   (b) $\displaystyle\int_0^{0.4} \sqrt{1+x^4} \, dx$ ($|\text{error}| < 5 \times 10^{-6}$)

Applications of Taylor Polynomials

6. Find the Taylor polynomial $T_3(x)$ for the function $f(x) = \cos x$ centered at $x = \dfrac{\pi}{2}$. Graph f and T_3 on the same screen.

7. (a) Approximate $f(x) = e^{x^2}$ by a Taylor polynomial with degree 3 at $x = 0$.
   (b) Use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_3(x)$ when $0 \le x \le 0.1$.

8. Use the information from Exercise 6 to estimate $\cos(80^\circ)$ correct to five decimal places.

9. Use Taylor's Inequality to determine the number of terms of the Maclaurin series for e^x that should be used to estimate $e^{0.1}$ to within 0.00001.

10. Use the Alternating Series Estimation Theorem or Taylor's Inequality to estimate the range of values of x for which the given approximation is accurate to within the stated error. Check your answer graphically.
    (a) $\sin x \approx x - \dfrac{x^3}{6}$ ($|error| < 0.01$)
    (b) $\arctan x \approx x - \dfrac{x^3}{3} + \dfrac{x^5}{5}$ ($|error| < 0.05$)