Week #7

Some problems and solutions selected or adapted from Stewart Calculus.

Series

1. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.
   (a) $3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$
   (b) $10 - 2 + 0.4 - 0.08 + \cdots$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$
   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}}$

2. Find the values of x for which the series converges. Find the sum of the series for those values of x.
   (a) $\displaystyle\sum_{n=1}^{\infty} (-5)^n x^n$
   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n}$

Alternating Series

3. Test the series for convergence or divergence.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$
   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$
   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$
   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} e^{2/n}$

4. Show that the series is convergent. How many terms of the series do we need to add in order to find the sum to the indicated accuracy?
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^6}$ $(|\text{error}| < 0.00005)$
   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{10^n n!}$ $(|\text{error}| < 0.000005)$

5. Is the 50th partial sum s_{50} of the alternating series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ an overestimate or underestimate of the total sum? Explain.

Representations of Functions as Power Series

6. Find a power series representation for the function and determine the interval of convergence.

$$f(x) = \frac{x}{9+x^2}$$

7. (a) Use differentiation to find a power series representation for

$$f(x) = \frac{1}{(1+x)^2}$$

   What is the radius of convergence?
   (b) Use part (a) to find a power series for

$$f(x) = \frac{1}{(1+x)^3}$$

   (c) Use part (b) to find a power series for

$$f(x) = \frac{x^2}{(1+x)^3}$$

8. Find a power series representation for the function and determine the radius of convergence.
   (a) $f(x) = \ln(5-x)$
   (b) $f(x) = \dfrac{x}{(1+4x)^2}$

9. Evaluate the indefinite integral as a power series. What is the radius of convergence?
   (a) $\displaystyle\int \frac{t}{1-t^3}\, dt$

(b) $\int x^2 \ln(1+x)\, dx$

10. Use a power series to approximate the definite integral below to six decimal places.

$$\int_0^{0.2} \frac{1}{1+x^5}\, dx$$
