

Week #4

Some problems and solutions selected or adapted from Stewart Calculus.

Gradients and the Directional Derivative

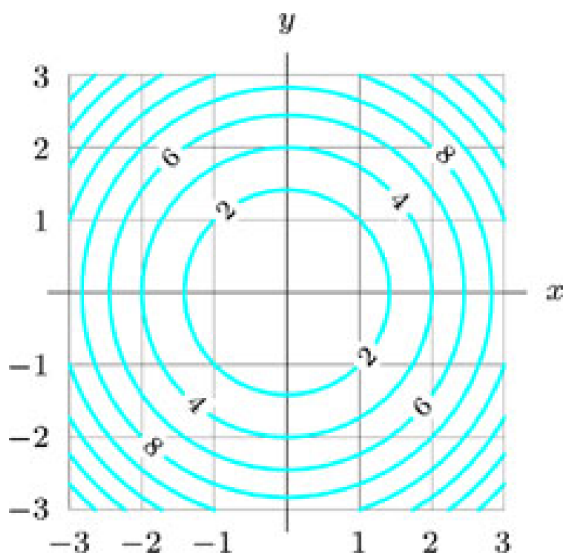
In Problems 1-5 find the gradient of the given function; if a point is also given, evaluate the gradient at that specific point. Assume the variables are restricted to a domain on which the function is defined.

1. $f(x, y) = \frac{3}{2}x^5 - \frac{4}{7}y^6$
2. $f(x, y, z) = 1/(x^2 + y^2 + z^2)$
3. $f(x, y, z) = xe^y \sin z$
4. $f(x, y) = \sqrt{x^2 + y^2}$
5. $z = \sin(x/y)$

In Problems 6-7, find the directional derivative $f_{\vec{u}}(1, 2)$ for the function f with $\vec{u} = \left\langle \frac{3}{5}, \frac{-4}{5} \right\rangle$.

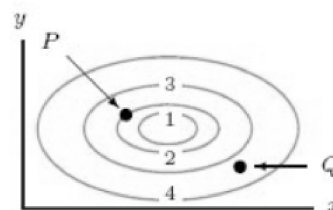
6. $f(x, y) = 3x - 4y$
7. $f(x, y) = xy + y^3$

In Problems 8-13, use the contour diagram of $f(x, y)$ shown below to decide if the specified directional derivative is positive, negative, or approximately zero.



8. At the point $(-2, 2)$, in direction $\langle 1, 0 \rangle$.
9. At the point $(0, -2)$, in direction $\langle 0, 1 \rangle$.
10. At the point $(-1, 1)$, in direction $\langle 1, 1 \rangle$.

11. At the point $(-1, 1)$, in direction $\langle -1, 1 \rangle$.
12. At the point $(0, -2)$, in direction $\langle 1, 2 \rangle$.
13. At the point $(0, -2)$, in direction $\langle 1, -2 \rangle$.
14. The contour diagram below represents the level curves $f(x, y)$.



In each of the following parts, decide whether the given quantity is positive, negative or zero. Explain your answer.

- (a) The value of $\nabla f \cdot \langle 1, 0 \rangle$ at P .
 - (b) The value of $\nabla f \cdot \langle 0, 1 \rangle$ at P .
 - (c) $\frac{\partial f}{\partial x}$ at Q .
 - (d) $\frac{\partial f}{\partial y}$ at Q .
15. The temperature at any point in the plane is given by the function

$$T(x, y) = \frac{100}{x^2 + y^2 + 1}$$

where the temperature is in $^{\circ}\text{C}$, and the positions are in metres.

- (a) What shape are the level curves of T ?
- (b) Where on the plane is it hottest? What is the temperature at that point?
- (c) Find the direction of the greatest increase in temperature at the point $(3, 2)$. What is the magnitude of that greatest increase (including units)?
- (d) Find the direction of the greatest decrease in temperature at the point $(3, 2)$.
- (e) Find a direction at the point $(3, 2)$ in which the temperature does not increase or decrease.