Week #3

Some problems and solutions selected or adapted from Stewart Calculus.

Tangent Lines and Intersections

1. Find an equation of the tangent plane to the given surface at the specified point.

   (a) $z = 3y^2 - 2x^2 + x$, $(2, -1, -3)$

   (b) $z = \sqrt{xy}$, $(1, 1, 1)$

2. Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3, 2, 6)$ and use it to approximate the number $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$.

3. If $z = 5x^2 + y^2$ and (x, y) changes from $(1, 2)$ to $(1.05, 2.1)$, compare the values of Δz and dz.

   Note: here Δz is the actual change in z using the equation above, and dz is the change using the linearization $L(x, y)$ at (1,2).

4. The length and width of a rectangle are measured as 30cm and 24cm, respectively, with an error in measurement of at most 0.1cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

5. If R is the total resistance of three resistors, connected in parallel, with resistances R_1, R_2, R_3, then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

   If the resistances are measured in ohms as $R_1 = 25\,\Omega$, $R_2 = 40\,\Omega$, $R_3 = 50\,\Omega$, with a possible error of 0.5% in each case, estimate the maximum error in the calculated value of R.

The Chain Rule

For questions **6-7** use the Chain Rule to find the indicated partial derivatives.

6. $z = x^4 + x^2 y, \quad x = s + 2t - u, \quad y = stu^2;$
   $\dfrac{\partial z}{\partial s}, \dfrac{\partial z}{\partial t}, \dfrac{\partial z}{\partial u}$ when $s = 4, t = 2, u = 1$

7. $w = xy + yz + zx, \quad x = r\cos\theta, \quad y = r\sin\theta, \quad z = r\theta$
   $\dfrac{\partial w}{r}, \dfrac{\partial w}{\theta}$ when $r = 2, \theta = \pi/2$

8. The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1 + t}$, $y = 2 + \dfrac{1}{3}t$, where x and y are measured in centimeters. The temperature function satisfies $T_x(2, 3) = 4$ and $T_y(2, 3) = 3$. How fast is the temperature rising on the bug's path after 3 seconds?

9. The pressure of 1 mole of an ideal gas is increasing at a rate of 0.05 kPa/s and the temperature is increasing at a rate of 0.15 K/s. Use the equation $PV = (8.31)T$ to find the rate of change of the volume when the pressure is 20 kPa and the temperature is 320 K.
