

Week #2

Some problems and solutions selected or adapted from Stewart Calculus.

Partial Derivatives

For questions 1-5, find the first partial derivatives of the function.

1. $f(x, t) = e^{-t} \cos \pi x$

2. $z = (2x + 3y)^{10}$

3. $f(x, y) = \frac{x}{y}$

4. $f(x, y, z) = xz - 5x^2y^3z^4$

5. $w = \ln(x + 2y + 3z)$ For questions 6-7, use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$.

6. $x^2 + 2y^2 + 3z^2 = 1$

7. $e^z = xyz$ For question 8, find all the second partial derivatives of the given function.

8. $z = \arctan\left(\frac{x+y}{1-xy}\right)$

Contour Diagrams and Partial Derivatives

9. To the right is the contour diagram for a surface $z = f(x, y)$. The contour lines are spaced at intervals of $\Delta z = 1$, and they increase as we move to the right. Also $f(0, 0) = 0$.

(a) Use the diagram to estimate the partial derivatives $f_x(1, 3)$ and $f_y(1, 3)$. For reference, the point $(1, 3)$ is at the cross-hairs on the contour diagram.

(b) Use the diagram to find the sign of the second partial derivatives

- $f_{xx}(1, 3)$,
- $f_{yy}(1, 3)$, and
- $f_{xy}(1, 3)$.

