APSC 172 Practice Problems

Week #1

Some problems and solutions selected or adapted from Stewart Calculus.

Equations of Lines and Planes

1. Find a parametric equation for the line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$.

2. Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

$$L_1 : \frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-1}{-3}$$

$$L_2 : \frac{x-3}{1} = \frac{y+4}{3} = \frac{z-2}{-7}$$

3. For each part below, find an equation of the plane matching the given description.

   (*a*) The plane through the point $(-1, \frac{1}{2}, 3)$ and with normal vector $\mathbf{i} + 4\mathbf{j} + \mathbf{k}$

   (*b*) The plane through the point $(1, -1, -1)$ and parallel to the plane $5x - y - z = 6$

   (*c*) The plane through the point $(1, \frac{1}{2}, \frac{1}{3})$ and parallel to the plane $x + y + z = 0$

   (*d*) The plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$

   (*e*) The plane that passes through the point $(-1, 2, 1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$

Slopes of Trajectories

4. We use the in-class definition of the slope of a 3D parametric curve as the z rise over the xy run of the velocity vector: $m = \dfrac{\dot{z}}{\sqrt{(\dot{x})^2 + (\dot{y})^2}}$.

   Consider a particle whose projected path on the xy plane follows $[x, y] = [2t, 3t - 1]$, as it passes over the surface defined by $z = x - y^2$.

   (a) Find the xyz location of the particle at $t = 2$, and then determine if the particle is moving uphill or downhill.

   (b) Find the xyz location of the particle at $t = -1$, and find the instantaneous slope at that time.