Week #7 - The Fundamental Theorem of Calculus

Some problems and solutions selected or adapted from Stewart Calculus and Hughes-Hallett Calculus-Early Transcendentals.

Definite Integrals in Modeling

1. The rate at which the world's oil is being consumed is continuously increasing. Suppose the rate of oil consumption (in billions of barrels per year) is given by the function $r = f(t)$, where t is measured in years and $t = 0$ is the start of 2004.

   (a) Write a definite integral which represents the total quantity of oil used between the start of 2004 and the start of 2009.

   (b) Suppose $r = 32e^{0.05t}$. Using a left-hand sum with five subdivisions, find an approximate value for the total quantity of oil used between the start of 2004 and the start of 2009.

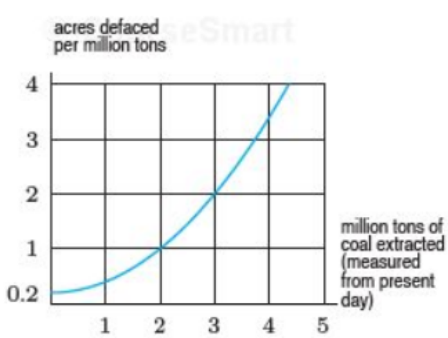
   (c) Interpret each of the five terms in the sum from part (b) in terms of oil consumption.

2. As coal deposits are depleted, it becomes necessary to strip-mine larger areas for each ton of coal. The graph below shows the number of acres of land per million tons of coal that will be defaced during strip-mining as a function of the number of million tons removed, starting from the present day.

   (a) Estimate the total number of acres defaced in extracting the next 4 million tons of coal (measured from the present day). Draw four rectangles under the curve, and compute their area.

   (b) Re-estimate the number of acres defaced using rectangles above the curve.

   (c) Combine your answers to parts (a) and (b) to get a better estimate of the actual number of acres defaced.



3. The following table gives the emissions, E, of nitrogen oxides, in millions of metric tons per year, in the US. Let t be the number of years since 1970 and $E = f(t)$.

   (a) What are the units and meaning of $\int_0^{30} f(t)\, dt$?

   (b) Estimate $\int_0^{30} f(t)\, dt$.

Year	1970	1975	1980	1985	1990	1995	2000
E	26.9	26.4	27.1	25.8	25.5	25.0	22.6

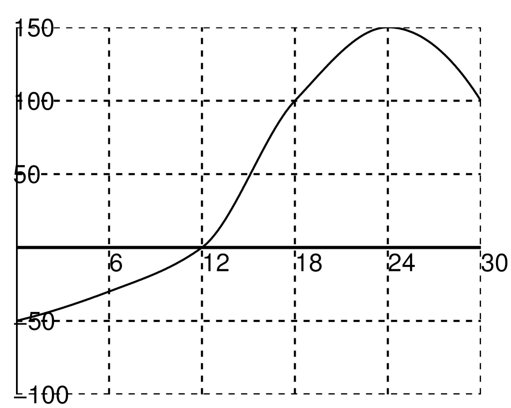
4. Coal gas is produced at a gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. The following measurements, made at the start of each month, show the rate at which pollutants are escaping (in tons/ month) in the gas:

Time (months)	0	1	2	3	4	5	6
Rate pollutants escape (tons/month)	5	7	8	10	13	16	20

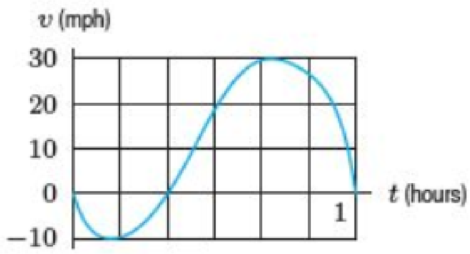
   (a) Make an overestimate and an underestimate of the total quantity of pollutants that escape during the **first month**.

   (b) Make an overestimate and an underestimate of the total quantity of pollutants that escape during the **six months** shown in the table.

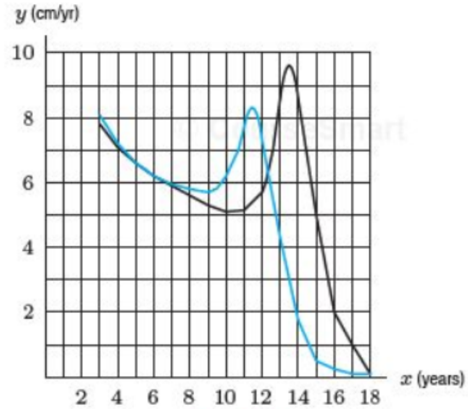
5. The graph below shows the rate of change of the quantity of water in a water tower, in liters per day, during the month of April. If the tower had 12,000 liters of water in it on April 1, estimate the quantity of water in the tower on April 30.



6. A bicyclist pedals along a straight road with velocity v given in the graph below. She starts 5 miles from a lake; positive velocities take her away from the lake and negative velocities take her toward the lake. When is the cyclist farthest from the lake, and how far away is she then?



7. Height velocity graphs are used by endocrinologists (doctors specializing in the study of hormones) to follow the progress of children with growth deficiencies. The graph below shows the height velocity curves of an average boy and an average girl between ages 3 and 18.



(a) Which curve is for girls and which is for boys? Explain how you can tell.

(b) About how much does the average boy grow between ages 3 and 10?

(c) The growth spurt associated with adolescence and the onset of puberty occurs between ages 12 and 15 for the average boy and between ages 10 and 12.5 for the average girl. Estimate the height gained by each average child during this growth spurt.

(d) When fully grown, about how much taller is the average man than the average woman? (The average boy and girl are about the same height at age 3.)

Integration By Anti-Derivatives

To practice computing anti-derivatives, do as many of the problems from below, **and from the textbook**, as you need.

Additional problems. Evaluate the following integrals.

8. $\int_{-1}^{2} (x^3 - 2x)dx$

9. $\int_{-1}^{1} x^{100}dx$

10. $\int_{1}^{4} (5 - 2t + 3t^2)dt$

11. $\int_{0}^{1} (1 + \frac{1}{2}u^4 - \frac{2}{5}u^9)du$

12. $\int_{1}^{9} \sqrt{x}dx$

13. $\int_{1}^{8} x^{-2/3}dx$

14. $\displaystyle\int_{\pi/6}^{\pi} \sin\theta d\theta$

15. $\displaystyle\int_{-5}^{5} e\ dx$

16. $\displaystyle\int_{0}^{1} (u+2)(u-3)du$

17. $\displaystyle\int_{0}^{4} (4-t)\sqrt{t}\ dt$

18. $\displaystyle\int_{1}^{9} \frac{x-1}{\sqrt{x}}dx$

19. $\displaystyle\int_{0}^{2} (y-1)(2y+1)dy$

20. $\displaystyle\int_{0}^{\pi/4} \sec^2 t\ dt$

21. $\displaystyle\int_{0}^{\pi/4} \sec\theta\tan\theta d\theta$

22. $\displaystyle\int_{1}^{2} (1+2y)^2 dy$

23. $\displaystyle\int_{0}^{3} (2\sin x - e^x)dx$

24. $\displaystyle\int_{1}^{2} \frac{v^3+3v^6}{v^4}dv$

25. $\displaystyle\int_{1}^{18} \sqrt{\frac{3}{z}}dz$

26. $\displaystyle\int_{0}^{1} (x^e + e^x)dx$

27. $\displaystyle\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}dx$

28. $\displaystyle\int_{1}^{2} \frac{4+u^2}{u^3}du$

29. $\displaystyle\int_{-1}^{1} e^{u+1}du$

30. $\displaystyle\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}}dx$

31. $\displaystyle\int_{0}^{\pi} f(x)dx \text{ where } f(x) = \begin{cases} \sin x & \text{if } 0 \le x < \pi/2 \\ \cos x & \text{if } \pi/2 \le x \le \pi \end{cases}$

32. $\displaystyle\int_{-2}^{2} f(x)dx \text{ where } f(x) = \begin{cases} 2 & \text{if } -2 \le x \le 0 \\ 4-x^2 & \text{if } 0 < x \le 2 \end{cases}$

33. If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t)dt$ represent?

34. The current in a wire is defined as the derivative of the charge: $I(t) = Q'(t)$. What does $\int_a^b I(t)dt$ represent?

35. If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time t, what does $\int_0^{120} r(t)dt$ represent?

36. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)dt$ represent?

37. The marginal revenue function $R'(x)$ is defined as the derivative of the revenue function $R(x)$, where x is the number of units sold. What does $\int_{1000}^{5000} R'(x)dx$ represent?

38. If $f(x)$ is the slope of a trail at a distance of x miles from the start of the trail, what does $\int_3^5 f(x)dx$ represent?

39. If x is measured in meters and $f(x)$ is measured in Newtons, what are the units for $\int_0^{100} f(x)dx$?

40. The linear density of a rod of length 4m is given by $\rho(x) = 9 + 2\sqrt{x}$ measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod.

41. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \le t \le 50$. Find the amount of water that flows from the tank during the first 10 minutes.

42. The velocity of a car was read from its speedometer at 10-second intervals and recorded in the table. Use the Midpoint Rule on 5 intervals to estimate the distance travelled by the car.

t(s)	v(mi / h)	t(s)	v(mi / h)
0	0	60	56
10	38	70	53
20	52	80	50
30	58	90	47
40	55	100	45
50	51		

43. Suppose that a volcano is erupting and readings of the rate $r(t)$ at which solid materials are spewed into the atmosphere are given in the table. The time t is measured in seconds and the units for $r(t)$ are tonnes (metric tons) per second.

t	0	1	2	3	4	5	6
$r(t)$	2	10	24	36	46	54	60

(a) Use the LEFT and RIGHT sums to give upper and lower estimates for the total quantity $Q(6)$ of erupted materials after 6 seconds.

(b) Use the Midpoint Rule to estimate $Q(6)$.

44. The marginal cost of manufacturing x yards of a certain fabric is $C'(x) = 3 - 0.01x + 0.000006x^2$ (in dollars per yard). Find the increase in cost if the production level is raised from 2000 yards to 4000 yards.