Week #2 - Vector-Valued Functions

Some problems and solutions selected or adapted from Stewart Calculus and Hughes-Hallett Calculus-Early Transcendentals.

Vector-Valued Functions

1. Find the domain of the vector function $\mathbf{r}(t) = \langle \sqrt{4-t^2}, e^{-3t}, \ln(t+1) \rangle$

2. Find the domain of the vector function $\mathbf{r}(t) = \left\langle \dfrac{t-2}{t+2},\ (\sin t),\ \ln(9-t^2) \right\rangle$

3. Sketch the curve of the vector equation $\mathbf{r}(t) = \langle \sin t, t \rangle$ and indicate with an arrow the direction in which t increases.

4. Sketch the curve of the vector equation $\mathbf{r}(t) = \langle t^3, t^2 \rangle$ and indicate with an arrow the direction in which t increases.

5. For $x = 3 - 4t, \quad y = 2 - 3t$

   a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

   b) Eliminate the parameter to find a Cartesian equation of the curve.

6. For $x = \sqrt{t}, \quad y = 1 - t;$

   a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

   b) Eliminate the parameter to find a Cartesian equation of the curve.

7. Consider the trajectory defined by $\mathbf{r}(t) = \langle 3\cos(3t), 2 + \cos^2(3t) \rangle$.

   (a) Eliminate the parameter from the trajectory to get the xy relationship in the trajectory.

   (b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

8. For $x = e^t - 1, \quad y = e^{2t}$,

   a) Eliminate the parameter to find a Cartesian equation of the curve.

   b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

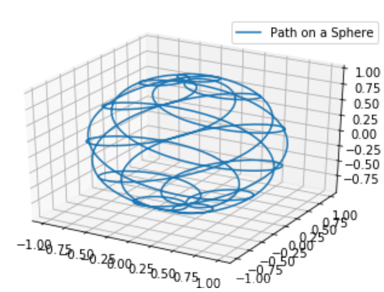
9. For $x = \cos\left(\dfrac{1}{2}\theta\right), \quad y = \sin\left(\dfrac{1}{2}\theta\right)$ and $-\pi \leq \theta \leq \pi$,

   (a) Eliminate the parameter to find a Cartesian equation of the curve.

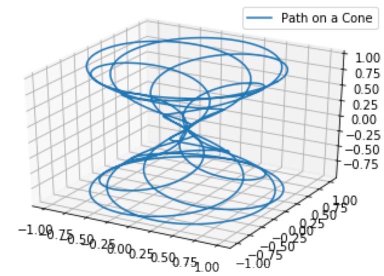
   (b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

10. Define a vector-valued function that traces out an ellipse centered at the origin, with an x-axis 'radius' of 7 and a y-axis 'radius' of 3.

11. Show that the trajectory defined by
    $\mathbf{u}(t) = \langle \sin(t)\cos(2.75t), \sin(t)\sin(2.75t), \cos(t) \rangle$
    lies on the unit sphere, defined by $x^2 + y^2 + z^2 = 1$.



12. Show that the trajectory defined by
$\mathbf{u}(t) = \langle \cos(t)\cos(2.75t), \cos(t)\sin(2.75t), \cos(t)\rangle$
lies on the cone defined by $x^2 + y^2 = z^2$.



13. Describe the motion of a particle with position (x, y) as t varies, given by $x = \sin\ t,\ \ y = \cos^2\ t,\ \ -2\pi \leq t \leq 2\pi$

14. Suppose the trajectories of two particles are given by the vector functions

$$\mathrm{r}_1(t) = \langle t^2, 7t - 12, t^2\rangle \qquad \mathrm{r}_2(t) = \langle 4t - 3, t^2, 5t - 6\rangle$$

for $t \geq 0$. Do the particles collide?

15. Two particles travel along the space curves

$$r_1(t) = \langle t^3, t, t^2\rangle \qquad r_2(t) = \langle 1 + 26t, 1 + 2t, 1 + 8t\rangle$$

Do the particles collide? Do their paths intersect?