

## Week #1 - Derivatives and Vectors

Some problems and solutions selected or adapted from Stewart Calculus and Hughes-Hallett Calculus-Early Transcendentals.

### Computing Derivatives

Below are a **small sample of problems** involving the computation of derivatives. They are **not enough** to properly learn and memorize how to apply all the derivative rules. You should practice with as many problems as you need to become proficient at computing derivatives.

Further practice problems can be found in any calculus textbook.

- Let  $f(x) = 4e^x - 9x^2 + 5$ . Compute  $f'(x)$ .
- Let  $f(x) = 2x^6\sqrt{x} + \frac{-5}{x^3\sqrt{x}}$ . Compute  $f'(x)$ .
- Let  $f(x) = \frac{7x^2 + 7x + 5}{\sqrt{x}}$ .
  - Compute  $f'(x)$ .
  - Find  $f'(3)$ .
- Let  $f(t) = 7t^{-7}$ .
  - Compute  $f'(t)$ .
  - Find  $f'(3)$ .
- Let  $f(x) = 4e^x + e^1$ . Compute  $f'(x)$ .
- Let  $f(x) = 4e^x + 4x$ . Compute  $f'(x)$ .
- $f(x) = (3x^2 - 2)(6x + 3)$ .
  - Compute  $f'(x)$ .
  - Find  $f'(4)$ .
- Let  $f(x) = \frac{\sqrt{x} - 4}{\sqrt{x} + 4}$ . Compute  $f'(9)$ .
- Consider  $f(x) = \frac{4x + 3}{3x + 2}$ .
  - Compute  $f'(x)$ .
  - Find  $f'(5)$ .
- Consider  $f(x) = \frac{7 - x^2}{7 + x^2}$ .
  - Compute  $f'(x)$ .
  - Find  $f'(1)$ .
- Let  $f(x) = -2x(x - 3)$ .
  - Compute  $f'(x)$ .
  - Find  $f'(-5)$ .
- $f(x) = \frac{4x^3 - 3}{x^4}$ .
  - Compute  $f'(x)$ .
  - Find  $f'(2)$ .
- $g(x) = \frac{e^x}{5 + 4x}$ . Compute  $g'(x)$ .
- $f(x) = \frac{4x^2 \tan x}{\sec x}$ .
  - Find  $f'(x)$ .
  - Find  $f'(3)$ .
- $f(x) = 7 \sin x + 12 \cos x$ .
  - Compute  $f'(x)$ .
  - Find  $f'(1)$ .
- Let  $f(x) = \cos x - 2 \tan x$ . Compute  $f'(x)$ .
- $f(x) = \frac{5 \sin x}{3 + \cos x}$ .
  - Compute  $f'(x)$ .
  - Find  $f'(2)$ .
- $f(x) = 7x(\sin x + \cos x)$ .
  - Compute  $f'(x)$ .
  - Find  $f'(3)$ .
- Let  $f(x) = \cos(\sin(x^2))$ . Compute  $f'(x)$ .
- Let  $f(x) = 2 \sin^3 x$ . Compute  $f'(x)$ .
- Let  $y = (8 + \cos^2 x)^6$ . Compute  $\frac{dy}{dx}$ .
- Let  $f(x) = -3 \ln[\sin(x)]$ . Compute  $f'(x)$ .
- Let  $f(x) = 2 \ln(4 + x)$ . Compute  $f'(x)$ .
- Let
$$P = \frac{V^2 R}{(R + r)^2}.$$
Calculate  $\frac{dP}{dr}$ , assuming that  $r$  is variable and  $R$  and  $V$  are constant.

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### Vector Basics

For Problems 25-29, say whether the given quantity could be represented only as a vector, or if a scalar could be used to represent it.

- The current population of Canada.
- The distance from Toronto to Vancouver.
- The magnetic field at a point on the earth's surface.
- The current temperature at a point on the earth's surface.
- The populations of each of the 13 provinces and territories.

For Problems 30-37, find the resulting vector, and write it as both (a) a sum of unit vectors  $\vec{i}$ ,  $\vec{j}$ ,  $\vec{k}$ , as well as (b) component-only form,  $\langle a, b, c \rangle$ .

30.  $-4\langle 1, -2, 0 \rangle - 0.5\langle 1, 0, -1 \rangle$
31.  $2\langle 0.45, -0.9, -0.01 \rangle - 0.5\langle 1.2, 0, -0.1 \rangle$
32.  $(3\vec{i} - 4\vec{j} + 2\vec{k}) - (6\vec{i} + 8\vec{j} - \vec{k})$
33.  $(4\vec{i} + 2\vec{j}) - (3\vec{i} - \vec{j})$
34.  $(\vec{i} + 2\vec{j}) + (-3)(2\vec{i} + \vec{j})$
35.  $-4(\vec{i} - 2\vec{j}) - 0.5(\vec{i} - \vec{k})$
36.  $2\langle 0.45\vec{i} - 0.9\vec{j} - 0.01\vec{k} \rangle - 0.5\langle 1.2\vec{i} - 0.1\vec{k} \rangle$
37.  $(4\vec{i} - 3\vec{j} + 7\vec{k}) - 2(5\vec{i} + \vec{j} - 2\vec{k})$

For Problems 38-41, perform the indicated operations on the following vectors:

$$\begin{aligned} \vec{a} &= \langle 0, 2, 1 \rangle, & \vec{b} &= \langle 3, 5, 4 \rangle \\ \vec{c} &= \langle 1, 6, 0 \rangle, & \vec{x} &= \langle 2, 9, 0 \rangle \\ \vec{y} &= \langle 4, -7, 0 \rangle, & \vec{z} &= \langle 1, -3, -1 \rangle \end{aligned}$$

38. Find  $\vec{a} + \vec{z}$ .
39. Find  $2\vec{c} + \vec{x}$ .
40. Find  $2\vec{a} + 7\vec{b} - 5\vec{z}$ .
41. Find  $\|\vec{y} - \vec{x}\|$ .
42. Find a vector with length 2 that points in the same direction as  $\vec{i} - \vec{j} + 2\vec{k}$ .

For Problems 43-46, consider the following scenario. A cat on the ground at the point  $(1, 4, 0)$  watches a squirrel at the top of a tree. The tree is one unit high with its base at  $(2, 4, 0)$ . Find the displacement vectors for the points described in Problems 43-46.

43. From the origin to the cat.
44. From the bottom of the tree to the squirrel.
45. From the bottom of the tree to the cat.
46. From the cat to the squirrel.

For Problems 47-51, find the length of the vectors given.

47.  $\vec{v} = \vec{i} - \vec{j} + 2\vec{k}$
48.  $\vec{v} = \vec{i} - 3\vec{j} - \vec{k}$
49.  $\vec{v} = \langle 1, -1, 3 \rangle$

$$50. \vec{v} = 7.2\vec{i} - 1.5\vec{j} + 2.1\vec{k}$$

$$51. \vec{v} = \langle 1.2, -3.6, 4.1 \rangle$$

52. For each of the four statements below, answer the following questions: Does the statement make sense? If yes, is it true for all possible choices of  $\vec{a}$  and  $\vec{b}$ ? If no, why not?

- (a)  $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
- (b)  $\vec{a} + \|\vec{b}\| = \|\vec{a} + \vec{b}\|$
- (c)  $\|\vec{b} + \vec{a}\| = \|\vec{a} + \vec{b}\|$
- (d)  $\|\vec{a} + \vec{b}\| = \|\vec{a}\| + \|\vec{b}\|$

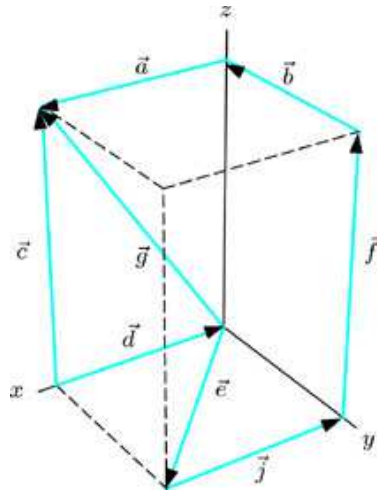
53. For what values of  $t$  are the following pairs of vectors parallel?

- (a)  $2\vec{i} + (t^2 + (2/3)t + 1)\vec{j} + t\vec{k}, 6\vec{i} + 8\vec{j} + 3\vec{k}$
- (b)  $t\vec{i} + \vec{j} + (t - 1)\vec{k}, 2\vec{i} - 4\vec{j} + \vec{k}$
- (c)  $2t\vec{i} + t\vec{j} + t\vec{k}, 6\vec{i} + 3\vec{j} + 3\vec{k}$

54. Find all vectors  $\vec{v}$  in 2 dimensions having  $\|\vec{v}\| = 5$  and for which the  $\vec{i}$ -component of  $\vec{v}$  is  $3\vec{i}$ .

55. The diagram below shows a rectangular box containing several vectors. Are the following statements true or false? Explain.

- (a)  $\vec{c} = \vec{f}$
- (b)  $\vec{a} = \vec{d}$
- (c)  $\vec{a} = -\vec{b}$
- (d)  $\vec{g} = \vec{f} + \vec{a}$
- (e)  $\vec{e} = \vec{a} - \vec{b}$
- (f)  $\vec{d} = \vec{g} - \vec{c}$



## The Dot Product, Perpendicular and Parallel

For Problems 56-63, perform the following operations on the given 3-dimensional vectors.

$$\vec{a} = \langle 0, 2, 1 \rangle, \quad \vec{b} = \langle -3, 5, 4 \rangle, \quad \vec{c} = \langle 1, 6, 0 \rangle$$
$$\vec{y} = \langle 4, -7, 0 \rangle, \quad \vec{z} = \langle 1, -3, -1 \rangle$$

56.  $\vec{a} \cdot \vec{y}$

57.  $\vec{c} \cdot \vec{y}$

58.  $\vec{a} \cdot \vec{b}$

59.  $\vec{a} \cdot \vec{z}$

60.  $\vec{a} \cdot (\vec{c} + \vec{y})$

61.  $\vec{c} \cdot \vec{a} + \vec{a} \cdot \vec{y}$

62.  $(\vec{a} \cdot \vec{b})\vec{a}$

63.  $(\vec{a} \cdot \vec{y})(\vec{c} \cdot \vec{z})$

64. Let  $\vec{v} = \langle 2, 3 \rangle$ . Using only two-dimensional vectors, find a unit vector in the same direction as  $\vec{v}$ , and then find another vector perpendicular to  $\vec{v}$ .

65. Which pairs (if any) of vectors from the following list

(a) Are perpendicular?

(b) Are parallel?

(c) Have an angle less than  $\pi/2$  between them?

(d) Have an angle of more than  $\pi/2$  between them?

$$\vec{a} = \langle 1, -3, -1 \rangle, \vec{b} = \langle 1, 1, 2 \rangle,$$

$$\vec{c} = \langle -2, -1, 1 \rangle, \vec{d} = \langle -1, -1, 1 \rangle$$

66. Which pairs of the vectors  $\sqrt{3}\vec{i} + \vec{j}$ ,  $3\vec{i} + \sqrt{3}\vec{j}$ ,  $\vec{i} - \sqrt{3}\vec{j}$  are parallel and which are perpendicular?

67. Compute the angle between the vectors  $\vec{i} + \vec{j} + \vec{k}$  and  $\vec{i} - \vec{j} - \vec{k}$ .

68. (a) Give a 2-dimensional vector that is parallel to, but not equal to,  $\vec{v} = \langle 4, 3 \rangle$ .

(b) Give a vector that is perpendicular to  $\vec{v}$ .

69. For what values of  $t$  are  $\vec{u} = \langle t, -1, 1 \rangle$  and  $\vec{v} = \langle t, t, -2 \rangle$  perpendicular? Are there values of  $t$  for which  $\vec{u}$  and  $\vec{v}$  are parallel?